PRINCIPLES FOR COMPUTING THE EFFICIENCY OF A SYSTEM WITH LOW-TEMPERATURE HEAT PIPES

G. F. Smirnov

UDC 536.248.2

An approach to the computation of the efficiency of a system with heat pipes is discussed. Conditions are analyzed for the axial transport of heat along the wall and framework of a heat pipe. A scheme to compute the thermal resistance of a heat pipe is recommended. Efficiency criteria are formulated.

The computational principle varies depending on the indicator by which the efficiency of a system with heat pipes is appraised. Computation of the efficiency of a system according to thermal engineering indices such as the temperature drop and energy consumption is discussed in this paper. In many cases these indices are governing in the solution of the question of the expediency of using heat pipes.

It will be shown below that central to the computation of the efficiency of a system with heat pipes is the computation of the total thermal resistance of the heat pipe.

Recommendations on the computation of the total thermal resistance of a heat pipe in the extensive literature on heat pipes are limited and do not include fundamental cases.

Computation of Heat-Pipe Characteristics

The general diagram of thermal resistances is represented in Fig. 1, and its corresponding formula is

$$R_\Sigma = \frac{1}{\dfrac{1}{R_a} + \dfrac{1}{R'_{ev} + R_v + R'_c}} + R_{ev} + R_c. \tag{1}$$

If the connection between the heat pipe and the remaining system elements is set up by means of the wall temperatures on the heat supply and heat removal sections T_{ev} and T_c, then

$$\frac{1}{R'_\Sigma} = \frac{1}{R_a} + \frac{1}{R'_{ev} + R_v + R'_c}. \tag{2}$$

The quantities R_{ev} and R_c are determined taking account of the specific thermal conditions between the heat pipe and the system objects according to known recommendations [2]. As a rule, the quantity R_v is small, with the exception of those cases when a vacuum is maintained in the operating heat pipe ($P \leq 0.1$ *a*T*a*).

In these cases R_v can be computed according to [3, 4] or other similar recommendations. The main contributions to R_Σ and R'_Σ are made by the quantities R'_{ev} and R'_c, and in a number of cases by R_a.

It is necessary to compute R'_{ev} and R'_c by means of the formulas

$$R'_{ev} = \frac{1}{\alpha_{ev} F'_{ev}} \tag{3}$$

$$R'_c = \frac{1}{\alpha_c F'_c}. \tag{4}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 28, No. 2, pp. 197-207, February, 1975. Original article submitted February 15, 1974.

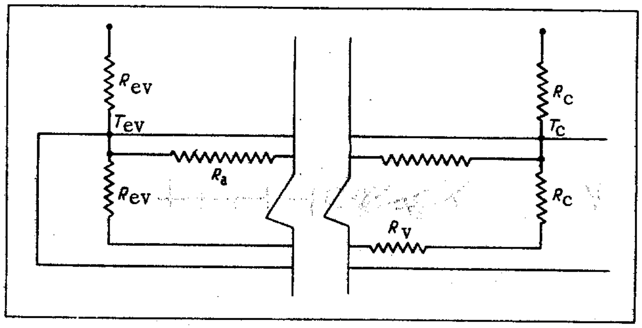

Fig. 1. Schematic diagram of the thermal resistances of a heat pipe.

The quantity α_{ev} depends essentially on the heat-transport mode. In the evaporation mode

$$\alpha_{ev} = \frac{\lambda_{ef}}{\delta_w} \qquad (5)$$

which is not suitable for "thick" wicks and small-diameter pipes. The cylindricity must be taken into account in a known manner in computing α_{ev}. There are relatively few test results on α_{ev} in the boiling modes. Existing correlations can be used only for those structures for which they have been obtained.

The coefficient of heat exchange α_c on the condensation section can be determined by means of (5) by substituting the quantities λ_{ef}, δ_w determined for the condensation section.

Contradictions exist in the recommendations for the computation of λ_{ef} [5-9]. For multilayer, reticular structures pressed to the heat-pipe wall λ_{ef} can be computed according to [5, 6]. For thin-layered reticular structures, as well as multilayer structures not pressed, [7] is preferable.

In conformity with the terminology of [10], the quantities F'_{ev} and F'_c can be called the "effective" evaporator and condenser surfaces, respectively. Since axial heat transport exists, $F'_{ev} > F_{ev}$ and $F'_c > F_c$ always. The latter is valid if the influence of the noncondensing gases is not taken into account. Otherwise,

$$F''_c = F'_c - \frac{2M_g R_g}{P_s d_v} \cdot \frac{d_0}{d_v}(T_s + T_c) \qquad (6)$$

should be substituted in place of F'_c in (4).

The mass of gas M_g is estimated from an analysis of the priming conditions or from experiment. Therefore, the correct determination of the resistances R'_{ev} and R'_c requires taking account of the influence of heat transfer on the "effective" dimensions of the evaporation and condensation sections in the general case.

For heat pipes with short adiabatic zones, the axial heat transport results not only in an increase in the evaporation and condensation surfaces, but also in the appearance of the resistance R_a, which is commensurate with the resistance of the loop $R'_{ev} + R_v + R'_c$.

A correct estimation of the role of axial heat transport in the general heat-exchange process in a heat tube is of substantial value not only for the computation of the efficiency of a cooling system with heat pipes, but also for the correct decoding of test data and the computation of the hydraulic losses in the heat-pipe structure.

This question is examined inadequately in the literature. The papers [9-11] are devoted to computations of the temperature fields for unregulated heat pipes.

Approximate one-dimensional models of the heat conduction in the capillary structure of a heat pipe are proposed in [10, 11]. A computation of the quantities F'_{ev} and F'_c according to [10, 11] is impossible.

An analytic solution for the two-dimensional conjugate problem of fluid heat conduction and filtration in a heat-pipe structure has been obtained in [9].

According to [9], the main quantity of vapor is condensed in the initial section of the condensation zone, which is commensurate with the thickness of the structure. Such a deduction is not verified by the

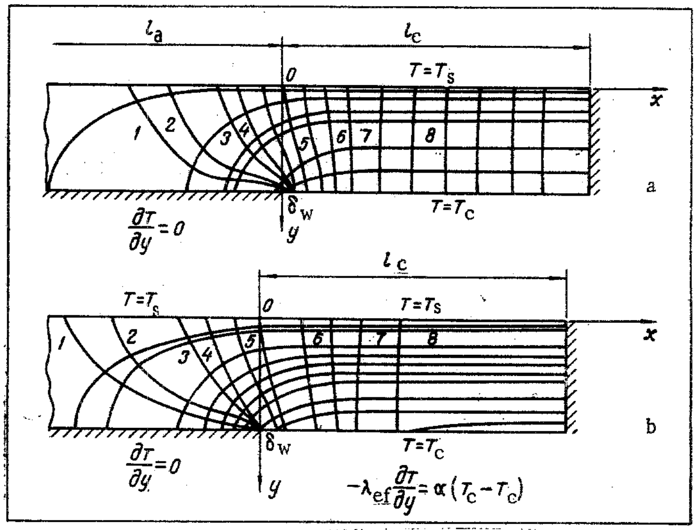

Fig. 2. Temperature field in a capillary heat-pipe structure for outer surface boundary conditions of the first (a) and third (b) kinds: 1) $Q_i/Q = 0.99$; 2) 0.98; 3) 0.95; 4) 0.90; 5) 0.85; 6) 0.75; 7) 0.50; 8) 0.40.

practice of heat-pipe operation. It is known that, other conditions being equal, the linear dimensions of the condensation section affect the operability of a heat pipe substantially. Analysis of the boundary conditions used in [9] showed the following.

1. The assumption about no mass condensation flux on the inner surface of the steam channel in the adiabatic zone contradicts physical reality. This is possible only when there is heat insulation on both the outside and inside of the adiabatic zone. In real heat-pipe constructions, only the outside of the adiabatic zone, but not the inside, can be considered heat-insulated. Hence, the corrected boundary conditions [9] will be

$$
\begin{gathered}
\text{for } y=0 \quad l_a \leqslant x \leqslant 0; \quad \frac{\partial P_e}{\partial y} = -kj; \\
l_c \geqslant x \geqslant 0; \quad P_{ei} = P_s - \sigma\left(\frac{1}{R_{1i}} + \frac{1}{R_{2i}}\right); \quad T = T_s; \\
\text{for } \quad y = \delta_w \quad l_c \geqslant x \geqslant 0; \quad T = T_c; \quad \frac{\partial P_e}{\partial y} = 0; \\
-l_a \leqslant x \leqslant 0; \quad \frac{\partial T}{\partial y} = 0; \quad \frac{\partial P_e}{\partial y} = 0; \\
\text{for } \quad x = l_c \quad 0 \leqslant y \leqslant \delta_w; \quad \frac{\partial T}{\partial x} = 0; \quad \frac{\partial P_e}{\partial x} = 0.
\end{gathered} \tag{7}
$$

2. The assumption about the existence of a condensation flux in the adiabatic zone adjacent to the condensation section and the approximately identical condensation rate along the length of the section agree with the solution of the filtration equation if the distribution of menisci is admitted along the length of the condensation zone.

3. If the wick thickness is assumed substantially greater than the dimension of the meniscus, then the problem cannot be solved as a conjugate problem, but can be solved separately. A. M. Roizengurt simulated the heat-conduction problem with boundary conditions (7) on a plane electrointegrator. Under actual conditions the wick thickness is substantially less than the diameter of the heat pipe. Roizengurt obtained the temperature fields in a heat-pipe wick for different δ_w/l relationships by electrical simulation on a plane EGDA 9/60 integrator.

One particular case is illustrated in Fig. 2. It is seen that the temperature distribution obtained on the boundary with the adiabatic zone in the formulation proposed for the constant condensation rate agrees qualitatively with experiment. The results of the electrical simulation are represented in Fig. 2 for the realization of boundary conditions of the third kind on the surface of the condensation section. The influence

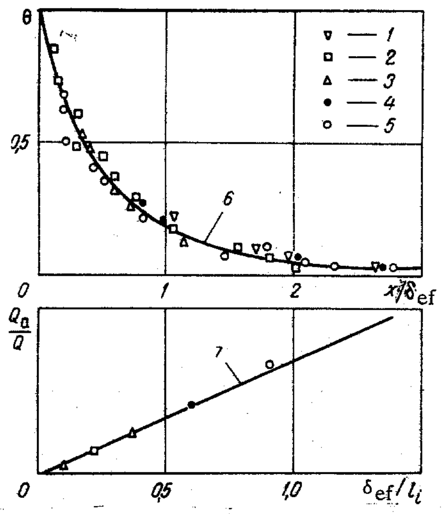

Fig. 3. Dependence of the temperature in the adiabatic zone of a heat pipe and the quantity of heat taken off on the geometry of the heat supply and elimination sections: 1) $\delta_{ef}/L_i = 0.053$; 2) 0.16; 3) 0.314; 4) 0.58; 5) 0.89; 6) $\Theta \cong \exp(-1.9\, x/\delta_{ef})$; 7) $Q_a/\cong 0.45\delta_{ef}/L_i$; $L_i = l_c$ or l_{ev}.

of the boundary conditions can be estimated qualitatively by means of the value of the number $Bi = \alpha\delta_w/\lambda_{ef}$.

Ordinarily, $Bi \ll 1$ for low-temperature heat pipes; hence, the heat-pipe surfaces can be considered isothermal on the evaporation and condensation section. For $Bi > 1$ the nonisothermy of the surface will influence the heat transfer. This must be taken into account, for example, according to [1]. Processing the electrical simulation results (Fig. 3) permitted generalized formulas to be obtained to take account of the axial heat transfer:

$$\Theta = \frac{T_s - T_a}{T_s - T_c} = \exp(-1.9\, x/\delta_{ef}), \tag{8}$$

$$Q_a \cong Q \cdot 0.45 \frac{\delta_{ef}}{l}. \tag{9}$$

Replacement of the true wick thickness δ_w by the equivalent thickness δ_{ef} permits taking account of the anisotropy of the heat transfer along the axis and along the normal to the wall. If the axial heat conductivity λ_0 is taken as reference value, then

$$\delta_{ef} = \delta_w \sqrt{\frac{\lambda_0}{\lambda_r}}. \tag{10}$$

Formulas (8) and (9) can be used for an approximate computation of the axial heat runoffs taking account of the influence of the wall. In this case

$$\lambda_0 = \frac{\lambda_{0w} f_w + \lambda_m f_m}{f_w + f_m}, \tag{11}$$

$$\lambda_r = \frac{1}{\dfrac{\delta_m}{\delta_m + \delta_w} \cdot \dfrac{1}{\lambda_m} + \dfrac{\delta_w}{\delta_w + \delta_m} \cdot \dfrac{1}{\lambda_{ef}}}, \tag{12}$$

$$\delta_{ef} = \sqrt{(\delta_m + \delta_w)\,\delta_w} \sqrt{\frac{\lambda_{0w}}{\lambda_{ef}} \cdot \frac{f_w}{f_w + f_m} + \frac{f_m}{f_w + f_m} \cdot \frac{\lambda_m}{\lambda_{ef}}}. \tag{13}$$

Assuming the "effective" lengths of the evaporation and condensation sections l'_{ev} and l'_c to be referred to the true lengths l_e and l_c just as the corresponding quantities of heat, it is possible to determine formulas for F'_{ev} and F'_c:

$$F'_{ev} = F_{ev}\left[1 + 0.45 \frac{\delta_{ef}}{L_{ev}}\right], \tag{14}$$

$$F'_c = F_c\left[1 + 0.45 \frac{\delta_{ef}}{L_c}\right]. \tag{15}$$

Formulas (8) and (9) are valid for a sufficiently long adiabatic zone. An analysis of (8) permits estimation of the length $(l_a)_{min}$ of the adiabatic zone for which the axial heat transfer can be neglected. If $l_a > (l_a)_{min}$, then

$$R_a \gg R'_{ev} + R_v + R'_c, \tag{16}$$

and R_a cannot be computed. For $l_a \le (l_a)_{min}$, we obtain by using the principle of superposition of temperature fields [2] for the adiabatic zone

$$T_x = T_s + (T_{ev} - T_s)\exp\left(-1.9 \frac{l_a - x}{\delta_{ef}}\right) - (T_{ev} - T_c)\exp\left(-1.9 \frac{x}{\delta_{ef}}\right). \tag{17}$$

For $l_a = 0$ we obtain the maximal axial heat flux by using (17), (8), and (9):

$$(q_a)_{max} \cong 0.45 \frac{\lambda_0}{\delta_{ef}} (T_{ev} - T_c). \tag{18}$$

The minimum resistance to heat transfer along the axis $(R_a)_{min}$

$$(R_a)_{min} = \frac{2.2}{\sqrt{\lambda_o \lambda_r}\, \pi d_o} \tag{19}$$

corresponds to this quantity.

Computations of $(R_a)_{min}$ by means of (19) show that in the overwhelming majority of cases

$$(R_a)_{min} \gg R'_{ev} + R_v + R'_c. \tag{20}$$

However, the quantity R_a can turn out to be commensurate with the sum $R'_{ev} + R_v + R'_c$ for short heat pipes or heat pipes working against gravity with a partly drained evaporation surface and a thick wall. Approximate formulas to compute the effective axial heat conduction without taking account of the mass-transfer process can be obtained from (18) and (19):

For $l_a = 0$

$$\lambda_{oef} = 1.8 \frac{l_o}{d_o} \sqrt{\lambda_o \lambda_r}. \tag{21}$$

For $l_a \gg \delta_{ef}$

$$\lambda_{oef} = 1.8 \frac{l_o + l_a}{d_o} \sqrt{\lambda_o \lambda_r} \frac{T_{ev} - T_c}{T_{ev} - T_c}. \tag{22}$$

$$l_o = 0.5\,(l_{ev} + l_c).$$

Comparing the values of λ_{ef} computed by means of (21) and (22) with test data

$$\lambda'_{ef} = \frac{4Q\,(l_a + l_o)}{\pi d_o^2\,(T_{ev} - T_c)}, \tag{23}$$

we can estimate the contribution of the axial heat conduction in the general heat-transfer mechanism in a heat pipe.

Not taken into account in the R_a computations was the heat-transfer component associated with the transfer of the supercooled fluid mass over the framework. Taking correct account of this factor is a complicated problem. Hence, attention was turned to the following considerations. If R_a is commensurate with $R'_{ev} + R_v + R'_c$, then heat transfer in the form of latent heat of phase transition is commensurate with the heat transferrable by heat conduction, and the heat of the supercooled fluid will be substantially less. Otherwise, this has no value.

The exception are cases when $[r/C_{ra}(T_{ev} - T_c)] \to 1$ (higher temperature drops and higher working pressures).

Therefore, by using (1)-(6), (14)-(16), (18)-(20), the thermal resistance of a heat pipe can be computed approximately.

General Statements on the Computation of the Thermal Engineering Efficiency of a System with Heat Pipes

Let us introduce the concept of the criterion of the relative efficiency of a cooling system with heat pipes K_z. The "content" of the criterion is determined by the approach which is proposed for the computation of the efficiency.

The crux of the approach consists of comparing two cooling systems (a cooling system with a heat pipe and a cooling system without a heat pipe) by means of some thermal engineering index Z_i (the thermal resistance, the energy expenditure, the pressure drop, etc.). The selection of this index is determined by the specifics of the system. Some fundamental cases will be examined below.

The computation of the criterion K_z is performed by a comparison with some quantity $(K_z)_{min}$.

Compliance with the inequality

$$K_z > (K_z)_{min} \tag{24}$$

is the foundation for the assertion about the expediency of using heat pipes in this system.

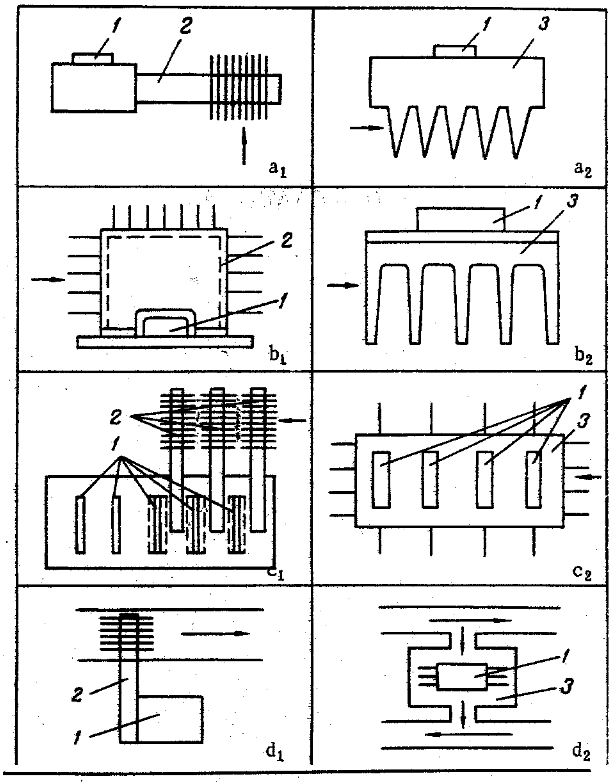

Fig. 4. Comparative diagrams of cooling systems with and without heat pipes: a_1) the heat-pipe-radiator 2, the object being cooled 1 is on the outer surface of the heat pipe; b_1) heat-pipe-radiator 2, the object being cooled 1 is within the heat pipe; c_1) heat-pipe heat-conductor 2 of a bulk source of heat evolution 1; d_1) heat-pipe heat-conductor 2 from the object being cooled 1 to the centralized cooling system; a_2, b_2, c_2, d_2) corresponding cooling systems without heat pipes; 3 are metal radiators.

If Z_{hp} is the value of the thermal engineering index in the system with the heat pipe by which the comparison is made, and Z_0 is the index in the system without the heat pipe, then

$$K_z = \frac{Z_o}{Z_{hp}}. \tag{25}$$

In the simplest case $(K_z)_{min} = 1$. However, if it is taken into account that an analysis of the relative efficiency of a heat pipe is approximate in nature, and that the use of heat pipes may not be logical for each positive effect but only for some minimum ΔZ_{min}, then

$$(K_z)_{min} = \frac{1}{1 - \frac{\Delta Z_{min}}{Z_o}}. \tag{26}$$

Let us note still another circumstance. An estimate of the expediency of using heat pipes according to the inequality (24) can be correct if the systems being compared are under almost optimal conditions. Hence, an optimization procedure should precede the computation of the criterion K_z.

Analysis of the Efficiency of Systems with Heat Pipes for Some Particular Cases

A. The Heat Pipe Is a Radiator of Part of a Radiator (Fig. 4a,b). The temperature drop can be chosen as the main thermal engineering index. Then $Z_{hp} = \Delta t_{hp}$, $\Delta t_{hp} = QR'_\Sigma$ for the diagram in Fig. 4a, and

$$Z_o = (1 - E_{ra})\,\Delta T_{ra},$$

$$K_z = \frac{(1 - E_{ra})\,\Delta T_{ra}}{\Delta t_{hp}}. \tag{27}$$

Use of a heat pipe in direct contact with the heat-evolving elements in Fig. 4b permits the reduction of the internal thermal resistance by a quantity Δt_v. Then

$$K_z = \frac{(1 - E_{ra})\,\Delta T_{ra} + \Delta t_v}{\Delta t_{hp}}. \tag{28}$$

B. The Heat Pipe Is an Internal Heat Drain in Bulky Heat-Evolving Modules (Fig. 4c). Just as in the first case, the pressure drop

$$Z_{hp} = \Delta t_{hp} + (T_{ce} - T_{ev}),$$
$$Z_o = T_{ef} - T_o; \quad K_z = \frac{T_{ce} - T_0}{\Delta t_{hp} + (T_{ce} - T_{ev})} \tag{29}$$

is taken as the comparison index.

C. The Heat Pipe Is a Heat Conductor from the Heat-Evolving Source to the Centralized Cooling System (Fig. 4d). If the temperature mode is taken as fixed, then the energy expenditures can be the thermal engineering index for the comparison, i.e.,

$$Z_{hp} = G_{hp}\Delta p_{hp}; \quad Z_o = G_o \Delta p_o;$$
$$K_z = \frac{G_o \Delta p_o}{G_{hp}\Delta p_{hp}}. \tag{30}$$

It can be assumed that the proposed approach to the computation of the relative efficiency is conserved even for other cases. Depending on the system operating conditions, mass-scale characteristics of a system, reliability, etc., can be used as indices for the comparison. If necessary, the efficiency can be estimated according to several indices, and in a first approximation the mean efficiency criterion

$$\overline{K} = \sum_{i=1}^{i=n} K_i g_i, \tag{31}$$

can be used, where g_i is the "weight" of the i-th index in a general analysis of the system characteristics. In the simplest case when the indices, the mass, and reliability, say, are equivalent (for weightless conditions)

$$K = \frac{\Sigma K_i}{n}.$$

The approximate scheme proposed for the computation of the relative efficiency of a system with heat pipes permits clarification of the domain of expedient application of heat pipes and a correct estimation of their advantages and disadvantages.

NOTATION

R_Σ	is the total thermal resistance of a heat pipe, including the contact resistances R_{ev} and R_c;
R_a	is the thermal resistance of axial heat transfer;
R_{ev} and R_c	are the thermal resistances of a heat pipe in contact with the objects of the system on the evaporation and condensation sections;
R'_{ev} and R'_c	are the radial (normal) internal thermal resistances of the heat pipe on the evaporation and condensation sections;
R_v	is the thermal resistance equivalent to a reduction of the temperature in the steam channel because of hydraulic drags;
R'_Σ	is the total thermal resistance of a heat pipe without taking R_{ev} and R_c into account;
λ_{ef}	is the coefficient of effective heat conduction of the wetted capillary structure of a heat pipe;
δ_w	is the wick thickness;
F_c and F_{ev}	are the surfaces of the heat-drain and heat-supply sections of a heat pipe;
M_g	is the mass of gas in the heat pipe;
R_g	is its gas constant;

p_s, T_s	are the vapor saturation pressure and temperature in the heat pipe;
d_0, d_v	are the inner diameters of the heat pipe and the vapor channel;
T_{ev}, T_c, and T_a	are the temperatures on the heat-supply and heat-drain sections and variable temperature in the adiabatic zone;
p_e	is the pressure in the fluid on the phase interface;
l_a, l_{ev}, l_c	are the lengths of the adiabatic, heat-supply, and heat-drain zones;
λ_{0w}, λ_r	are the coefficient of effective heat conduction in the structure along and normal to the wall of the heat pipe;
λ_0	is the mean coefficient of effective heat conduction taking account of the wall along the heat-pipe axis;
λ_m, δ_m	are the coefficient of heat conduction and thickness of the wall metal;
f_w, f_m	are the cross-sectional areas of the structure and the wall;
x, y	are the coordinates along the heat-pipe axis and along the normal to the wall;
λ_{ef}	is the effective coefficient of axial heat conduction referred to the total cross section of the heat pipe;
λ'_{0ef}	is the total effective coefficient of heat conduction of the heat pipe, referred to its total cross section;
Q	is the total quantity of heat transferred by the heat pipe, including on the adiabatic section Q_a;
Δt_{hp}	is the temperature drop in the heat pipe;
ΔT_{ra}, E_{ra}	are the temperature drop and efficiency of a radiator replacing the heat pipe;
Δt_{ev}	is the temperature drop between the surface of the object being cooled and the wall of the heat pipe;
T_{ce}	is the maximum temperature at the center of the object being cooled;
T_0	is the temperature on the surface of the object being cooled;
G_{hp}, Δp_{hp}	are the coolant discharge and hydraulic drag on the heat-drain section of the heat pipe;
G_0, Δp_0	are the same, on the surface of the object;
p_{ei}, R_{1i}, R_{2i}	are the pressure and principal radii of curvature on the surface of the i-th fluid layer;
σ	is the coefficient of surface tension on the vapor — fluid boundary;
j	is the mass flow of fluid normal to the surface of porous structure of the heat pipe;
k	is the proportionality coefficient between the pressure gradient in the porous structure and the mass flow under the assumption of correctness of the Darcy law.

LITERATURE CITED

1. W. Binert, "Use of heat pipes for temperature regulation," in: Heat Pipes [Russian translation], Mir, Moscow (1972).
2. G. N. Dul'nev and E. M. Semyashkin, Heat Exchange in Radio Electronic Apparatus [in Russian], Énergiya (1968).
3. L. L. Vasil'ev and S. V. Konev, "Heat and mass exchange in heat pipes," in: Heat and Mass Exchange at Low Temperatures [in Russian], Minsk (1970).
4. V. Ya. Sasin, M. I. Verba, and V. D. Portnov, "Physical conditions of heat and mass transfer in a heat pipe," in: Reports to a Scientific-Technical Conference at the Moscow Power Institute, 1968-1969. Industrial Thermal Power Section [in Russian].
5. G. F. Smirnov and L. N. Mishchenko, "Method of selecting the geometric parameters of low-temperature heat pipes," Teploénergetika, No. 8 (1973).
6. G. F. Smirnov and L. N. Mishchenko, "Investigation of the heat- and mass-transfer processes in low-temperature heat pipes," in: Heat and Mass Transfer [in Russian], Vol. 2, Part. 2, Minsk (1972).
7. V. G. Voronin, A. V. Revyakin, V. S. Tarasov, and V. Ya. Ustinov, "Investigation of a horizontal heat pipe," Kholod. Tekh., No. 9 (1972).
8. M. I. Verba, V. Ya. Sasin, and A. Ya. Shelginskii, "Physical heat-transfer conditions and analysis of heat pipes in the evaporation mode of operation in the moderate temperature range," Inzh.-Fiz. Zh., 23, No. 4 (1972).
9. F. A. Lyman and J. S. Huang, "Analysis of the temperature distribution in heat-pipe wicks," in: Heat Pipes [Russian translation], Mir, Moscow (1972).
10. K. H. Sun and C. L. Tien, "Simple conduction model for theoretical steady-state heat-pipe performance," AIAA J., 10, No. 8 1972.

11. G. F. Smirnov and L. N. Mishchenko, "Approximate method of computing the temperature field of a heat pipe," Questions of Radio Electronics [in Russian], Seriya TRTO, No. 2 (1972).